國立清華大學數學系訪問學者學術演講
NTHU Department of Mathematics
Visiting Scholar Colloquium

講題　　Iterated Galois groups of $X^2 + c$ over quadratic number fields with odd class number

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地點　　Room 723, General Building III

Abstract

Consider quadratic number fields $K = \mathbb{Q}(\sqrt{d})$, and the polynomial $f(X) = X^2 + c \in \mathbb{Z}[\sqrt{d}]$. Denote the n-th iteration of $f(X)$ by $f^n(X)$, then the Galois group $\mathrm{Gal}(f^n(X)/K)$ can be embedding into $[C_2]^n$ which is the n-th fold wreath product of cyclic group with 2 elements. We will give some criteria on the constant terms of $f^n(X)$ to determine that when the embedding is surjective for all $n \in \mathbb{N}$.

For the main results in the second part, we will focus on $K = \mathbb{Q}(\sqrt{d})$, where

1. $d = 2, q$ 　　　　q is a prime congruent to 1 modulo 4;
2. $d = 2, 2\ell, \ell\ell'$ 　ℓ, ℓ' are primes congruent to 3 modulo 4;
3. $d = -2, -\ell$ 　　ℓ is a prime congruent to 3 modulo 4.

All the cases have odd class numbers. We will use the quadratic residue properties for the fundamental unit to give some sufficient conditions on c such that $\mathrm{Gal}(f^n(X)/K) \cong [C_2]^n$.